COMPARISON OF OBJECTIVE FUNCTIONS FOR ENGINE MOUNTS OPTIMIZATION

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ABSTRACT

Engine's vibration propagating through its suspension is often responsible for low frequency noise perceived by vehicle occupants. In the design process and at early stage of development, it is useful to have an efficient way of estimating the properties of an engine suspension since these properties directly dictate the engine's structureborne vibration contribution to total noise of a vehicle. These properties include stiffness, position, orientation and number of engine mounts. A good design will provide adequate support of the engine as well as low power transmitted to the structure in order to reduce as much as possible the transmitted structureborne vibration and the radiated acoustic energy.

The purpose of this paper is to present an efficient and straightforward approach to develop and optimize engine suspension. Various objective functions such as transmitted forces, power flow injected in the elastic base structure, vibration levels taken at critical locations and driver ears acoustic pressure levels can be used. Using Matlab™, a dedicated and user-friendly software has been developed to assist and guide suspension designers. It allows the user to model each of the different parts of the system either as rigid bodies or as flexible structures .

NOMENCLATURE

x,y,z	translational degrees of freedom
$\theta_x, \theta_y, \theta_z$	rotational degrees of freedom
N	total number of degrees of freedom
n	total number of engine mounts
m	total number of external forces and moments
ω; f	frequency of vibration (in rad.s^{-1}; Hz)
i	$\sqrt{-1}$
$[\,]^T$	Transpose
$[M]$	Mass matrix
$[K]$	Complex stiffness matrix
η_r	structural damping loss factor of r^{th} mode

Subscript

p	external force or moment number
s	engine mounts number
g	refers to the center of gravity

1 INTRODUCTION

Passenger's comfort is of prime importance in nowadays vehicle design. In order to improve comfort, engine suspension design has to be based on comfort criteria describing passenger's perceptions. Acoustic pressure in vehicle cabin and vibration of components in contact with passengers (seats, driving wheel, floor) are typical examples. Using such an approach differs greatly from classical optimization techniques which only consider force injected into a rigid base structure. Considering only force functions limits the power of optimization techniques. In fact, passengers may not even feel the changes in perceptions between different configurations.

Furthermore, considering base structure as being rigid supposes that base stiffness is significantly higher from that of engine mounts. However, in certain circumstances, mounts stiffness and structure impedance may coincide and significantly alter engine's response to a determined excitation. In fact, at frame natural frequencies and for local mode, the mounts stiffness can even be higher than the attachment point frame stiffness. When the engine mount and frame coincide, a maximum power is transmitted to the frame.

Finally, transfer functions between force injected to the frame at engine mount locations and cabin vibration response or acoustic pressure often show high level peaks at specific frequencies. Cabin comfort can be compromise if there is energy transmitted at these frequencies from the powerplant. For all these reasons, it is appropriate to adopt a design method which takes into account both structure flexibility and transfer path between engine attachment points and passenger's zone. The substructuring approach is well suited for these requirements.

Classical optimization techniques usually minimize a cost function for only one driving condition, typically idling. This can lead to undesired increase in noise and vibration levels at other driving condition. This paper presents an optimization method which considers every steady state operating conditions of the engine.

2 THEORETICAL BACKGROUND

2.1 General assumptions

This paper presents a straightforward method to model the behavior of an elastically supported engine attached to a rigid or a flexible structure. Model predicts sound pressure or vibration level that are directly linked with passenger's perception using a so called comfort criterion. Each criterion is the sum in space and frequency of a cost function, and optimization is based on these criteria. The studied cost functions are:

- Force injected into a rigid structure
- Power injected into a flexible structure
- Vibration in passenger's zone
- Acoustic pressure in passenger's zone

The engine is modeled as a rigid body. External force load acting on the engine's body is referred to as shaking forces, moments and torques [Norton]. This periodic load is due to the inertia of the moving parts. At first, idling conditions are studied since external excitation frequencies and the system's natural frequencies are much closer than in any other conditions. In a second step, a whole RPM range will be considered since higher order modes might be ignored in the idle condition.

Mounts are made of neoprene which dissipates energy and offers a wide range of possible stiffness. Base is considered rigid or flexible depending on its stiffness relationship with mounts. All calculations are made in steady state conditions using a frequency based analysis. Figure 1 presents a schematic view of the system to model. For clarity purposes, structure is not shown.

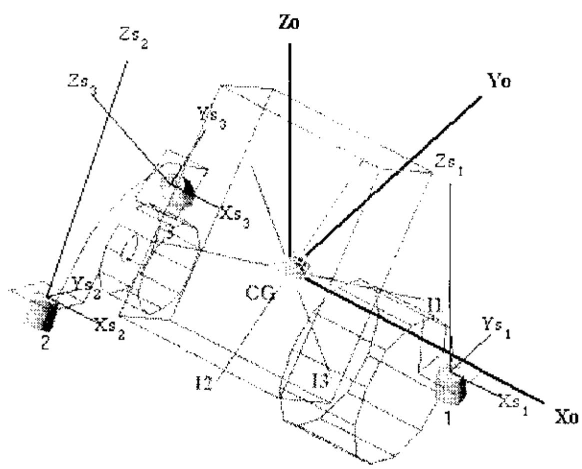

Figure 1 – Engine model with global coordinate system (CG,Xo,Yo,Zo), mounts coordinate system (i,Xsi,Ysi,Zsi) and principal axis of inertia (CG,I1,I2,I3).

2.2 Equations of motion

The engine is considered as a rigid body of mass *m*. Its center of gravity (*CG*), mass moments of inertia (*Ixx, Iyy...*) and principal inertia axes are known properties. The engine sits on *n* mounts with position *x,y,z* and orientation γ, τ relative to global coordinate system (CG,Xo,Yo,Zo) as illustrated in Figure 1. Since mounts are generally made of elastomeric material, dynamic complex stiffness is used to model hysteresis damping properties. All mounts properties expressed in local coordinate system (n_i,Xs_i,Ys_i,Zs_i) must be transformed into global coordinates system. Once engine and suspension are defined, the magnitude of the force applied to either rigid or flexible base structure can be estimated. These forces are then used to estimate several comfort criteria using the appropriate measured FRF.

A simple way to implement this approach is to combine a substructuring approach [1] which assembles all components into a global system and a rigid body model [2]. Governing equation for substructuring is :

$$\{f_s\} = \left(\left[H_{ss}^A\right] + \left[H_{ss}^B\right] + \left[K\right]^{-1}\right)^{-1}\left[H_{sp}^A\right]\{f_p\} \qquad (1)$$

where

$\{f_s\}$	Reaction forces in the connection points between engine and structure
$\left[H_{ss}^A\right]$	Compliance FRF matrix of the engine in free-free conditions at connection points
$\left[H_{ss}^B\right]$	Compliance FRF matrix of the base structure in free-free conditions at connecting points
$\left[K\right]$	Stiffness matrix containing the stiffness characteristics of the isolation elements
$\left[H_{sp}^A\right]$	Compliance FRF matrix describing transfer in free-free conditions from excitation points to interface points
$\{f_p\}$	Input forces and moments at excitation points

Most of the preceding FRF can be measured. The only FRF that can not be measured easily is [H_{sp}^A]. This FRF can be derived from the rigid body relations between excitation forces and interface points acceleration. Rewriting Heyns [2] equations in substructuring approach format, the following approach is obtained :

$$\left[H_{sp}^A\right] = \begin{bmatrix} \left[H_{11}^A\right] & \left[H_{1m}^A\right] \\ \left[H_{n1}^A\right] & \left[H_{nm}^A\right] \end{bmatrix} \qquad (2)$$

where an example of inner matrix for mount number 3 and force number 2 is defined as

$$\left[H_{32}^A\right] = \left[G_{3K}\right]\left[H_K\right]\left[G_{K2}\right] \qquad (3)$$

$$[G_{3g}] = \begin{bmatrix} 1 & 0 & 0 & 0 & z_s & -y_s \\ 0 & 1 & 0 & -z_s & 0 & x_s \\ 0 & 0 & 1 & y_s & -x_s & 0 \end{bmatrix}_{s=} \quad (4)$$

$$[H_g] = [M]^{-1} = \begin{bmatrix} m & 0 & 0 & 0 & 0 & 0 \\ 0 & m & 0 & 0 & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{XoXo} & I_{XoYo} & I_{XoZo} \\ 0 & 0 & 0 & I_{XoYo} & I_{YoYo} & I_{YoZo} \\ 0 & 0 & 0 & I_{XoZo} & I_{YoZo} & I_{XoZo} \end{bmatrix}^{-1} \quad (5)$$

$$[G_{g2}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -z_p & y_p & 1 & 0 & 0 \\ z_p & 0 & -x_p & 0 & 1 & 0 \\ -y_p & x_p & 0 & 0 & 0 & 1 \end{bmatrix}_{p=} \quad (6)$$

It is possible to determine force $\{f_s\}$ injected into rigid or flexible structure from eq. (1). This force is used to evaluate objective functions.

3 OBJECTIVE FUNCTIONS DEFINITION

3.1 Force injected in rigid structure

For a rigid structure, $\left|H^B_{ss}\right| \approx 0$, and eq. (1) can be rewritten as :

$$\{f_s\} = \left([H^A_{ss}] + [K]^{-1}\right)^{-1} [H^A_{sp}] \{f_p\} \quad (7)$$

where $\{f_s\}$ corresponds to the objective function. In order to minimize this function, a criterion φ_F is defined. This criterion corresponds to the total force injected into a rigid base.

3.2 Power injected in structure

Power injected in a flexible structure is defined as [3]

$$P = \frac{1}{2} \mathrm{Re}\left\{f_s^* \cdot j\omega [H^B_{ss}] \cdot f_s\right\} \quad (8)$$

where P corresponds to the objective function. In order to minimize this function, a criterion φ_{Pw} is defined. This criterion corresponds to the total power injected into a flexible base.

3.3 Acceleration at driver seat

Measured FRFs allow the computation of vibration levels in passenger's area. It is defined as :

$$\{a\} = [H^B_{as}]\{f_s\} \quad (9)$$

where $\{a\}$ corresponds to the objective function. In order to minimize this function, a criterion φ_{acc} is defined. This criterion corresponds to the total acceleration level of a specified region.

3.4 Acoustic pressure to the driver's ears

Acoustic pressure may be computed using :

$$\{P_r\} = [H^B_{ps}]\{f_s\} \quad (10)$$

where $\{Pr\}$ corresponds to the objective function. In order to minimize this function, a criterion φ_{Pr} is defined. This criterion corresponds to the total pressure level over all locations considered as described in [4]

4 OPTIMIZATION

Let {X} be a vector of mount properties such as position, orientation and stiffness. It is necessary to minimize φ with respect to variable {X}. Implementation of this optimization is done with Matlab's Optimization Toolbox functions. In order to run the optimization, upper and lower limits on {X} as well as constraints such as maximum engine displacement must be defined. Since the objective functions can be expressed in terms of single or multiple RPM values, the optimization Toolbox will solve for the specified RPM range in the objective function.

5 CASE STUDY

5.1 System description

Consider an IC engine attached to a flexible structure by means of 3 engine mounts. The engine's rigid body properties are tabulated in Figure 2.

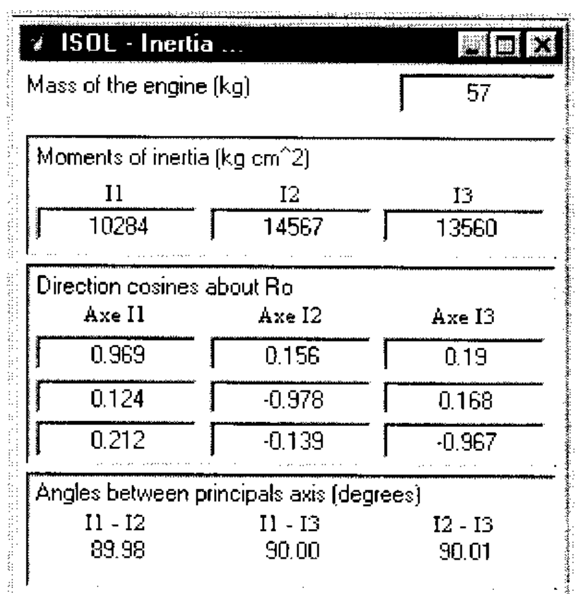

Figure 2 - Engine's rigid body properties

Engine mounts are made of neoprene and their axial (Z_s direction) and radial (X_s,Y_s direction) properties are shown in Figure 3. Both static (k_a) and dynamic ($k_a(f)$) stiffness have been measured. Static stiffness is only used to monitor static deflection response.

No	1	2	3
Active	1	1	1
Type	4	4	4
Ka(f) (N/mm)	2100	2100	2100
Kr(f) (N/mm)	460	460	460
Eta a (%)	15	15	15
Eta r (%)	15	15	15
Xo (cm)	23	-26	-26
Yo (cm)	3	-16	10
Zo (cm)	-7	-8	-8
Gamma (deg)	0	-30	30
Tau (deg)	0	0	0
Ka (N/mm)	966	966	966
Kr (N/mm)	125	125	125

Figure 3 – Engine mounts properties

Original configuration as illustrated in Figure 1 yields natural modal response shown in Figure 4.

ISOL - Modal prop. of engine on mounts

Natural frequencies

Mode	1	2	3	4	5	6
Freq (Hz)	12.3	12.9	16.6	31.2	37.5	54.0
Eta (%)	15.0	15.0	15.0	15.0	15.0	15.0

Mode shapes

Xo	0.19	0.46	0.02	0.01	0.00	0.00
Yo	0.24	0.29	0.04	0.01	0.06	0.00
Zo	0.00	0.01	0.00	0.52	0.01	0.04
Roll (Xo)	0.40	0.08	0.97	0.05	0.40	0.14
Pitch (Yo)	0.09	0.11	0.12	0.85	0.08	0.99
Yaw (Zo)	0.86	0.83	0.19	0.07	0.91	0.04

Figure 4 – Natural frequencies and mode shapes of engine on its mounts.

Shaking forces and torques for the whole range of operating conditions are shown in Figure 5.

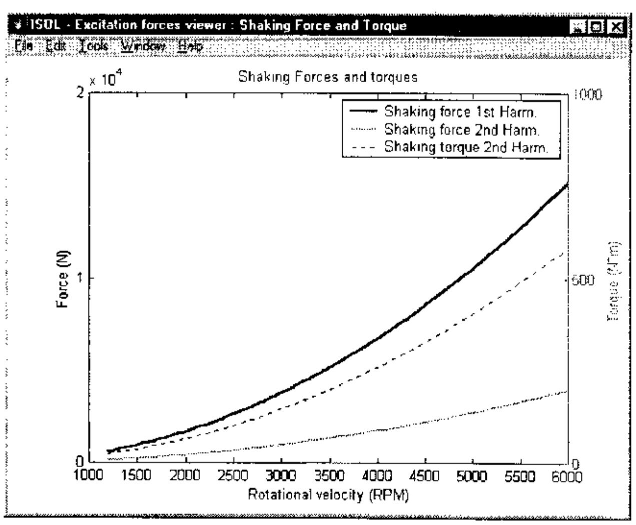

Figure 5 – Shaking forces and torques from inertia of moving parts

Figure 6 to Figure 9 present typical mobility (a/F) FRFs measured on vehicle.

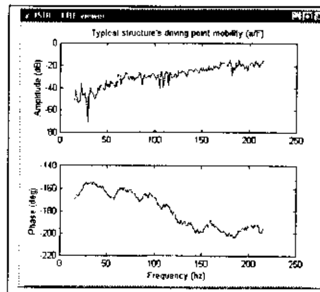

Figure 6 – Driving point FRF

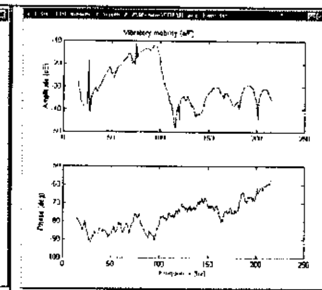

Figure 7 – Driver's seat FRF

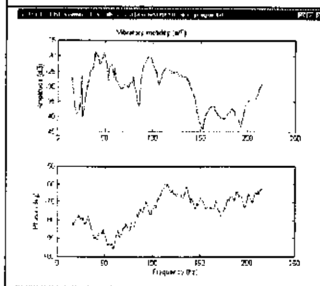

Figure 8 – Driving wheel FRF

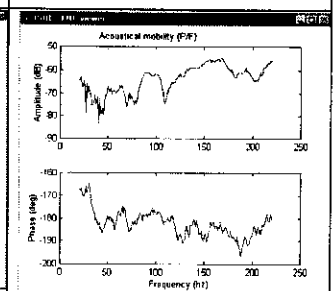

Figure 9 – Driver's ear FRF

5.2 Preliminary results

For simplicity and ease of interpretation, only positions of engine mounts are optimized. These mounts can move according to the geometry of the engine. In general, each mount may be moved up to 10 cm away from its original position.

5.2.1 Optimization based on idle condition

In this case, the comfort criterion φ_{acc} with acceleration point on steering wheel is minimized. Figure 10 presents φ_{acc} before and after optimization.

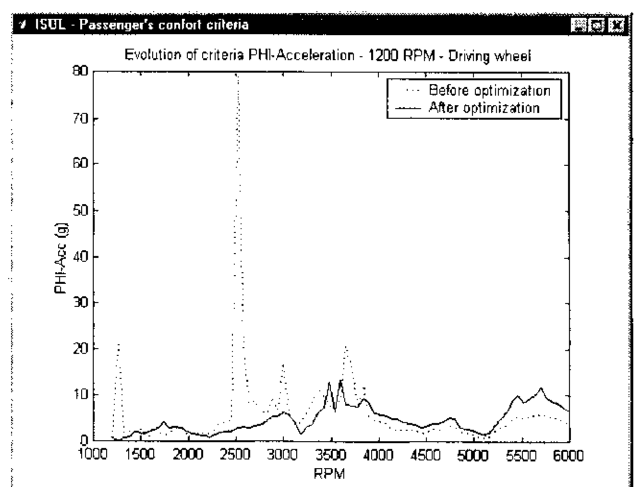

Figure 10 – Example of idling condition optimization

As expected, φ_{acc} is minimized at 1260 RPM and at its first harmonic frequency. This is not the case for most of the other driving conditions. Figure 10 must be compared with figure 11 which shows the result of an optimization on the same criterion but for 1200 to 6000 RPM.

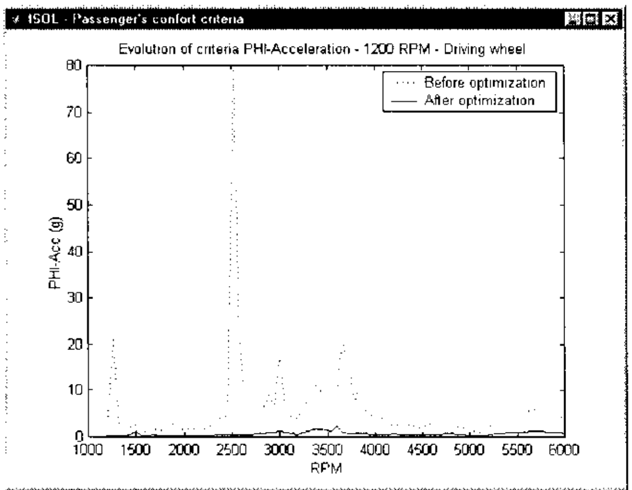

Figure 11 – Example of complete RPM range optimization

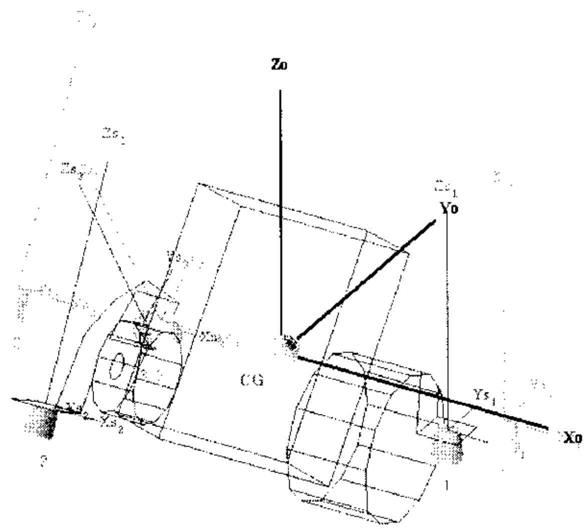

Figure 12 – Optimized position of engine mounts for case shown in Figure 11

In theory, optimizing on a wide range of RPM gives better results although it is more time consuming to complete the calculations. Since the optimization procedure tries to minimize the sum of PHI-ACC over the RPM range, intermediate spectrum might show great variation from the initial condition. It is important to properly set the termination tolerance in order to reach a reasonable minimization values. Figure 13 shows a minimization process over a complete RPM range. This case needed 232 function evaluations to reach this low level.

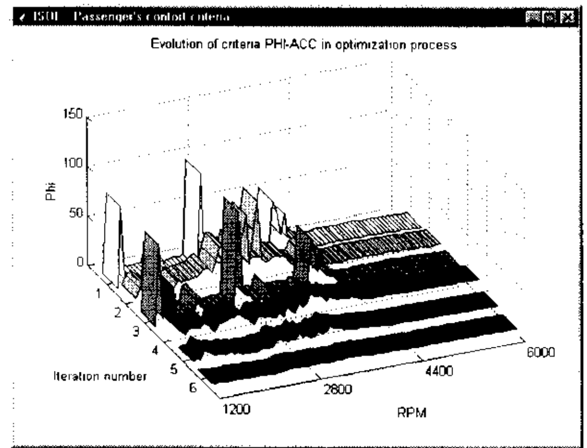

Figure 13 – Evolution of PHI-ACC over RPM range

5.2.2 *Optimization based on range 1200-6000 RPM*

In the next figures, results from optimization for a wide range of RPM are presented. These results suggest that from the starting configuration, each objective function taken separately can be well minimized. In fact, this shows that a significant gain of comfort might be obtained by changing the positions of the engine mounts.

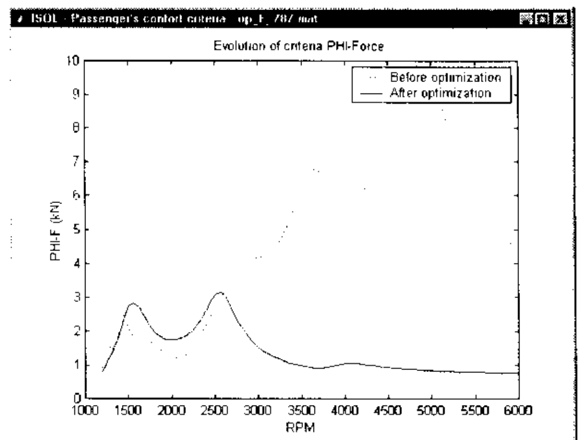

Figure 14 – Force criterion for force optimization

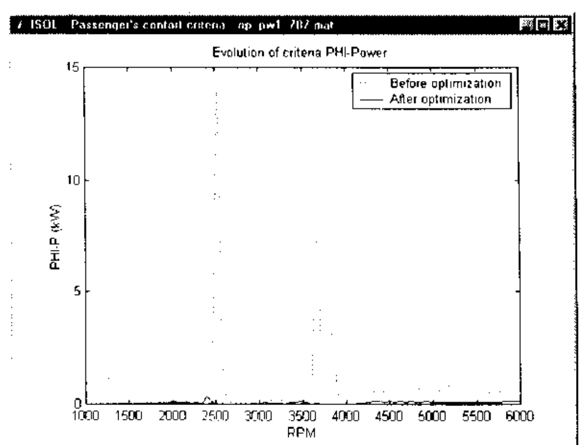

Figure 15 - Power criterion for power optimization

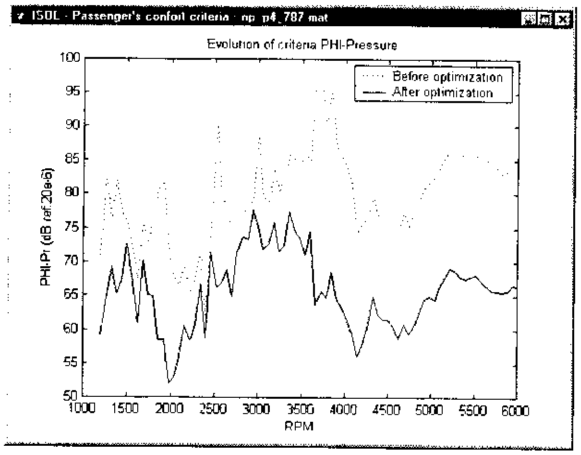

Figure 16 – Pressure criterion for pressure optimization

One should realize that optimization based on one criterion will not necessarily provide a good minimization of the other criteria. In other words, minimizing the power criterion will not necessarily minimize the force criterion. Figure 17 shows the level of minimization of the force applied to the rigid base when minimizing the power function. These results can not be generalized since they greatly depend on vehicle FRFs, suspension availability and engine type.

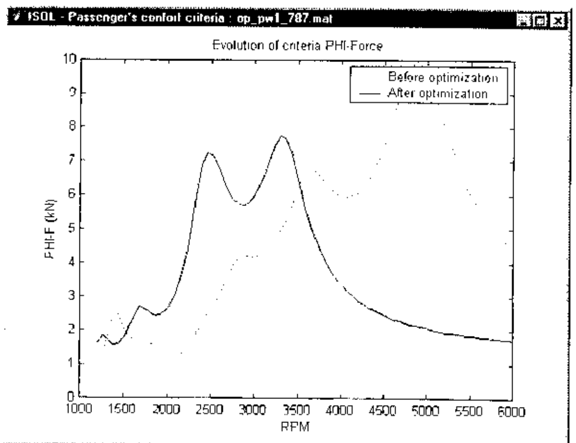

Figure 17 – Force criterion for power optimization

Finally, in order to find the best suitable engine mount configuration, it is possible to combine certain criteria as defined before, and to find the ideal position to minimize a weighted sum of criteria according to the desired NVH vehicle quality target.

6 CONCLUSION

An integrated approach in suspension optimization has been developed. It seeks to minimize the passenger's perception of noise and vibration inside the vehicle. It allows designers to model structureborne noise generated by the engine. That is from the point of excitation on the engine chassis, throughout the vehicle transfer path, and up to the vehicle cabin. It was demonstrated that optimizing with flexible base structure, adequate structure FRFs, and on a complete range of RPM is an efficient and promising approach. Optimization of the original position of the case study confirms that better positions can be found in order to enhance passengers comfort.

7 REFERENCES

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